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H_∞ Filtering of 2-D FM LSS Model with State Delays*

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Abstract: In this paper, we deal with H_∞ filtering problem for a class of two-dimensional (2-D) discrete time-invariant systems with state delays described by local state-space (LSS) Fornasini-Marchesini (FM) second model. Based on the bounded real lemma of 2-D state-delayed systems, H_∞ filtering design is developed, such that the filtering error system is asymptotically stable and has H_∞ performance γ via LMIs' feasibility. Furthermore, the minimum H_∞ norm bound γ can be obtained by solving a linear objective optimization problem. A numerical example is given to demonstrate the effectiveness and advantage of our result.

Key Words: 2-D state-delayed systems, H_∞ disturbance attenuation, H_∞ filtering, Linear matrix inequality (LMI)

1 INTRODUCTION

Over the past several decades, a considerable interest has been devoted to problems involving multi-variable (m-V) systems and multi-dimensional (m-D) signals, most of which were expressed as two-dimensional (2-D) discrete-system models [1]. Among the major results developed so far concerning the 2-D signals and systems (for example, robust stability [2, 3], H_∞ control [4] and guaranteed cost control [5]), the estimation of the state variables for 2-D dynamic systems using available noisy measurements is a fundamental problem in signal processing, image processing and control, so it have received significant attention and various Lyapunov approaches have been proposed as effective tools in the past two decades (see, e.g., [6-8], and the references cited therein). One of the celebrated approaches was Kalman (H_2) filter [7]. The Kalman type of estimation requires two fundamental assumptions: the availability of an exact internal of a system and a priori information on the external noises (like white noise, etc.). However, a practical system is difficult to satisfy the assumptions, so current efforts on the topic of 2-D signal estimation are mainly H_∞ filtering approach. Compared with the Kalman filter, the advantage of the H_∞ filtering is that the noise sources can be arbitrary signals with bounded energy, or bounded average power instead of being Gaussian. So H_∞ filtering tends to be more robust when there exist additional parameter disturbances in models and it is very appropriate in a number of practical situations [6]. In view of known and unknown statistical characteristics of noise input, it is often required that the designed filter makes the studied system satisfy multiplicate performances, for example, H_2 performance. Therefore, robust mixed H_2/H_∞ filtering for 2-D systems with polytopic uncertainties was studied in [8] using a less conservative parameter-dependent Lyapunov function approach [9].

The existence of delays is frequently a source of instability. Much work has been reported on the problem of the stability of standard, often termed 1-D in the m-D systems literature, linear systems with delays see, e.g., [10]. The need for 2-D stability and stabilization problems is motivated by practical relevance of 2D discrete linear systems with delays. Consider, for example, the case of linear repetitive processes [11], which arise in the modeling of industrial processes such as material rolling [12]. Based on practical physical background, stability and H_∞ control problems of 2-D state-delayed systems were considered in [13-15], respectively. So far, most results for the 2-D filtering problem focus on systems without delays, though for specific H_∞ filtering design considered in reference [16].

In this paper, we study H_∞ filtering problem for 2-D state-delayed systems. We design a 2-D filter with a general form to guarantee the filtering error system has H_∞ disturbance attenuation γ, and the minimum H_∞ norm bound γ can be obtained by solving a linear optimization problem using LMIs. Finally, it is shown that, via a numerical example, the proposed methodology is feasible. The filtering design procedure is less conservative than that obtained in [16].

2 H_∞ PERFORMANCE ANALYSIS

Consider a class of 2-D discrete systems with different state delays in different directions expressed by FM LSS model proposed by Fornasini and Marchesini [17]:

$$\begin{aligned} x(i+1,j+1) &= A_1x(i+1,j)+A_2x(i,j+1) \\ &\quad +A_{1d}x(i+1,j-d_1) \\ &\quad +A_{2d}x(i-d_2,j+1) \\ &\quad +B_1\omega(i+1,j)+B_2\omega(i,j+1) \quad (1) \\ z(i,j) &= Lx(i,j)+L_1\omega(i,j) \quad (2) \end{aligned}$$

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where $x(i,j) \in R^n$ is the local state vector, $\omega(i,j) \in R^m$ is the noise signal, which are assumed to be unknown, i.e., $\|\omega\|_2 < \infty, z(i,j) \in R^p$ is the control output. d_1 and d_2 are constant positive scalars representing delays along vertical direction and horizontal direction, respectively. $A_k, A_{kd}, B_k (k = 1,2), L$ and L_1 are constant matrices with appropriate dimensions.
The boundary conditions are assumed as:

$$\begin{aligned} \{x(i,j) = \varphi_{i,j}\}, \forall i &\geqslant 0, j = -d_1, -d_1+1, \cdots, 0 \\ \{x(i,j) = \psi_{i,j}\}, \forall j &\geqslant 0, i = -d_2, -d_2+1, \cdots, 0 \\ \varphi_{0,0} &= \psi_{0,0} \end{aligned} \quad (3)$$

and the transfer function of 2-D state-delayed system (1),(2) in channel $\omega \to z$ is given as:

$$\begin{aligned} G(z_1,z_2) &= L(z_1 z_2 I - A_1 z_1 - A_2 z_2 - A_{1d} z_1^{-d_1} \\ &\quad - A_{2d} z_2^{-d_2})^{-1}(B_1 z_1 + B_2 z_2) + L_1 \end{aligned} \quad (4)$$

The H_∞ performance measure for 2-D system (1),(2) with zero boundary conditions ($\varphi_{i0} = \psi_{0j} = 0$) is defined as follows.

Definition 1 (Paszke et al. [15]) 2-Ddiscrete linear state-delayed system described by (1),(2) with zero boundary conditions is said to have delay-independent (delay-dependent) H_∞ disturbance attenuation γ if it is asymptotically stable and

$$\|z\|_2 < \gamma \|\omega\|_2 \quad (5)$$

where $z = [\ z^{\mathrm{T}}(i+1,j), \quad z^{\mathrm{T}}(i,j+1)\ ]^{\mathrm{T}}, \omega = [\omega^{\mathrm{T}}(i+1,j), \omega^{\mathrm{T}}(i,j+1)]^{\mathrm{T}}$ and the l_2-norm of 2-D discrete signal z and ω are defined as

$$\begin{aligned} \|z\|_2^2 &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left(\|z(i+1,j)\|_2^2 + \|z(i,j+1)\|_2^2\right) \\ \|\omega\|_2^2 &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left(\|\omega(i+1,j)\|_2^2 + \|\omega(i,j+1)\|_2^2\right) \end{aligned}$$

For presentation convenience, we denote

$$\begin{aligned} A &= [\ A_1, \quad A_2\ ], A_d = [\ A_{1d}, \quad A_{2d}\ ] \\ B &= [\ B_1, \quad B_2\ ], L_d = diag\{L, L\} \\ L_{1d} &= diag\{L_1, L_1\} \end{aligned}$$

The following Corollary 1 provides a sufficient condition of H_∞ disturbance attenuation γ for 2-D state-delayed system (1),(2), upon which a H_∞ filter will be developed in Section III.

Lemma 1 (Paszke et al. [15]) 2-D state-delayed system (1),(2) with zero boundary conditions has H_∞ disturbance attenuation $\gamma > 0$ if there exist matrices $P > 0, Q_k > 0 (k = 1,2,3)$ such that the following matrix inequality holds:

$$\left[\begin{array}{ccc} A^{\mathrm{T}}PA - R_1 + L_d^{\mathrm{T}}L_d & A^{\mathrm{T}}PA_d & L_d^{\mathrm{T}}L_{1d} + A^{\mathrm{T}}PB \\ A_d^{\mathrm{T}}PA & A_d^{\mathrm{T}}PA_d - R_2 & A_d^{\mathrm{T}}PB \\ B^{\mathrm{T}}PA + L_{1d}^{\mathrm{T}}L_d & B^{\mathrm{T}}PA_d & L_{1d}^{\mathrm{T}}L_{1d} - \gamma^2 I + B^{\mathrm{T}}PB \end{array}\right] < 0 \quad (6)\ (7)$$

where $R_1 = diag\{P - Q_1 - Q_2 - Q_3, Q_1\}, R_2 = diag\{Q_2, Q_3\}$.

Remark 1 Lemma 1 is said to be bounded real lemma of 2-D state-delayed systems and the same as Theorem 2 in [16].

3 THE DESIGN OF H_∞ FILTER

In this section, we resolve the H_∞ filtering problem for 2-D state-delayed systems through designing a H_∞ filter, which makes the system have H_∞ disturbance attenuation γ. The design of H_∞ filter can be came down to solve a LMI.
Now, we consider a class of 2-D state-delayed systems in the form of

$$\begin{aligned} x(i+1,j+1) &= A_1 x(i+1,j) + A_2 x(i,j+1) \\ &\quad + A_{1d} x(i+1, j-d_1) \\ &\quad + A_{2d} x(i-d_2, j+1) \\ &\quad + B_1 \omega(i+1,j) + B_2 \omega(i,j+1) \quad (8) \\ y(i,j) &= Cx(i,j) + D\omega(i,j) \quad (9) \\ z(i,j) &= Lx(i,j) \quad (10) \end{aligned}$$

where $y(i,j) \in R^q$ is the measure output, C and D are constant matrices with appropriate dimensions. The boundary conditions of system (8),(10) are the same as (3).

Assumption 1 2-D system (8) is asymptotically stable when $\omega(i,j) = 0$.

Remark 2 Assumption 1 is made based on the fact that there is no control in the system (8) and it is a prerequisite for the filtering error system given below to be asymptotically stable. The filtering design is to examine whether it could track the original system very well, i.e. it guarantees the system's stability, based on any external disturbances, and a preselected H_∞ performance when there are external disturbances in the stable system (8).

Introduce a 2-D filter in the general form described by

$$\begin{aligned} \widehat{x}(i+1,j+1) &= A_{f_1}\widehat{x}(i+1,j) + A_{f_2}\widehat{x}(i,j+1) \\ &\quad + B_{f_1} y(i+1,j) + B_{f_2} y(i,j+1) \quad (11) \\ \widehat{z}(i,j) &= C_f \widehat{x}(i,j) + D_f y(i,j) \quad (12) \end{aligned}$$

with $\widehat{x}(0,0) = \widehat{x}(1,0) = \widehat{x}(0,1) = 0$, where $\widehat{x}(i,j) \in R^n, A_{f_k} \in R^{n\times n}, B_{f_k} \in R^{n\times q}(k=1,2), C_f \in R^{p\times n}$,and $D_f \in R^{p\times q}$.
Then, the filtering error system is expressed as:

$$\begin{aligned} \overline{x}(i+1,j+1) &= \overline{A}_1\overline{x}(i+1,j) + \overline{A}_2\overline{x}(i,j+1) \\ &\quad + \overline{A}_{1d}\overline{x}(i+1,j-d_1) \\ &\quad + \overline{A}_{2d}\overline{x}(i-d_2,j+1) \\ &\quad + \overline{B}_1\omega(i+1,j) + \overline{B}_2\omega(i,j+1) \quad (13) \\ \overline{z}(i,j) &= z(i,j) - \widehat{z}(i,j) \\ &= \overline{C}\overline{x}(i,j) + \overline{D}\omega(i,j) \quad (14) \end{aligned}$$

where

$$\begin{aligned} \overline{x}(i,j) &= [\ x^{\mathrm{T}}(i,j) \quad \widehat{x}^{\mathrm{T}}(i,j)\ ]^{\mathrm{T}} \\ \overline{x}(i,j-d_1) &= [\ x^{\mathrm{T}}(i,j-d_1) \quad \widehat{x}^{\mathrm{T}}(i,j-d_1)\ ]^{\mathrm{T}} \\ \overline{x}(i-d_2,j) &= [\ x^{\mathrm{T}}(i-d_2,j) \quad \widehat{x}^{\mathrm{T}}(i-d_2,j)\ ]^{\mathrm{T}} \end{aligned}$$

$$\begin{aligned} \overline{A}_k &= \begin{bmatrix} A_k & 0 \\ B_{f_k}C & A_{f_k} \end{bmatrix}, \overline{A}_{kd} = \begin{bmatrix} A_{kd} & 0 \\ 0 & 0 \end{bmatrix} \\ \overline{B}_k &= \begin{bmatrix} B_k \\ B_{f_k}D \end{bmatrix}, \overline{C} = [\ L - D_f C \quad -C_f\ ] \\ \overline{D} &= -D_f D (k = 1,2) \end{aligned}$$

and the transfer function of the filtering error system (13),(14) in channel $\omega \rightarrow \overline{z}$ is expressed as:

$$\begin{aligned}\overline{G}(z_1,z_2) &= \overline{C}(z_1z_2I-\overline{A}_1z_1-\overline{A}_2z_2-\overline{A}_{1d}z_1^{-d_1}\\ &\quad -\overline{A}_{2d}z_2^{-d_2})^{-1}(\overline{B}_1z_1+\overline{B}_2z_2)+\overline{D} \end{aligned} \tag{15}$$

Accordingly, the boundary conditions are assumed as:

$$\begin{aligned}\overline{x}(i,j) &= \{\varphi_{i,j}^{\mathrm{T}},0\}^{\mathrm{T}}, \forall i \geqslant 0, j=-d_1,-d_1+1,\cdots,0\\ \overline{x}(i,j) &= \{\psi_{i,j}^{\mathrm{T}},0\}^{\mathrm{T}}, \forall j \geqslant 0, i=-d_2,-d_2+1,\cdots,0\\ \varphi_{0,0} &= \psi_{0,0}\end{aligned} \tag{16}$$

Our objective is to develop a 2-D filter of the form (11),(12) such that the filtering error system (13),(14) has H_∞ disturbance attenuation γ. The design is obtained from the following theorem.

Theorem 1 2-D state-delayed system (8)-(10) with the boundary conditions (16) has H_∞ disturbance attenuation γ under the action of the filter (11),(12) if there exist matrices $X>0, \widetilde{Y}>0, Z, \overline{Z}_k, \widehat{Z}_k$ $(k=1,2)$ and D_f such that the following LMI holds:

$$\left[\begin{array}{ccccccccc} -J_P+\sum_{k=1}^{3}J_{Q_k} & 0 & J_{A_1}^{\mathrm{T}} & 0 & 0 & J_C^{\mathrm{T}} & 0 & 0 & 0\\ 0 & -J_{Q_1} & J_{A_2}^{\mathrm{T}} & 0 & 0 & 0 & J_C^{\mathrm{T}} & 0 & 0\\ J_{A_1} & J_{A_2} & -J_P & J_{A_{1d}} & J_{A_{2d}} & 0 & 0 & J_{B_1} & J_{B_2}\\ 0 & 0 & J_{A_{1d}}^{\mathrm{T}} & -J_{Q_2} & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & J_{A_{2d}}^{\mathrm{T}} & 0 & -J_{Q_3} & 0 & 0 & 0 & 0\\ J_C & 0 & 0 & 0 & 0 & -I & 0 & \overline{D} & 0\\ 0 & J_C & 0 & 0 & 0 & 0 & -I & 0 & \overline{D}\\ 0 & 0 & J_{B_1}^{\mathrm{T}} & 0 & 0 & \overline{D}^{\mathrm{T}} & 0 & -\gamma^2 I & 0\\ 0 & 0 & J_{B_2}^{\mathrm{T}} & 0 & 0 & 0 & \overline{D}^{\mathrm{T}} & 0 & -\gamma^2 I \end{array}\right] < 0 \tag{17}$$

where

$$\begin{aligned} J_P &= \left[\begin{array}{cc} X & \widetilde{Y}\\ \widetilde{Y} & \widetilde{Y}\end{array}\right], J_{B_k} = \left[\begin{array}{c} XB_k+\overline{Z}_kD\\ \widetilde{Y}B_k\end{array}\right]\\ J_{A_k} &= \left[\begin{array}{cc} XA_k+\overline{Z}_kC & XA_k+\overline{Z}_kC+\widehat{Z}_k\\ \widetilde{Y}A_k & \widetilde{Y}A_k\end{array}\right]\\ J_{A_{kd}} &= \left[\begin{array}{cc} XA_{kd} & XA_{kd}\\ \widetilde{Y}A_{kd} & \widetilde{Y}A_{kd}\end{array}\right](k=1,2)\\ J_C &= \left[\begin{array}{cc} L-D_fC & L-D_fC+Z\end{array}\right]\end{aligned}$$

Moreover, if the robust H_∞ filtering problem is solvable, the system matrices of H_∞ filter (11),(12) can be obtained as

$$\begin{aligned} A_{fk} &= (\widetilde{Y}-X)^{-1}\widehat{Z}_kB_{fk}=(\widetilde{Y}-X)^{-1}\overline{Z}_k\\ C_f &= -Z, D_f=D_f(k=1,2)\end{aligned} \tag{18}$$

Proof Suppose the H_∞ filtering problem is solvable, i.e. H_∞ filter (11),(12) makes the filtering error system (13),(14) have H_∞ disturbance attenuation γ, then it is derived from Lemma 1 that

$$\left[\begin{array}{cccc} -P+\sum_{k=1}^{3}Q_k & 0 & \overline{A}_1^{\mathrm{T}}P & 0\\ 0 & -Q_1 & \overline{A}_2^{\mathrm{T}}P & 0\\ P\overline{A}_1 & P\overline{A}_2 & -P & P\overline{A}_{1d}\\ 0 & 0 & \overline{A}_{1d}^{\mathrm{T}}P & -Q_2\\ 0 & 0 & \overline{A}_{2d}^{\mathrm{T}}P & 0\\ \overline{C} & 0 & 0 & 0\\ 0 & \overline{C} & 0 & 0\\ 0 & 0 & \overline{B}_1^{\mathrm{T}}P & 0\\ 0 & 0 & \overline{B}_2^{\mathrm{T}}P & 0 \end{array}\right. \tag{19}$$

$$\left.\begin{array}{ccccc} 0 & \overline{C}^{\mathrm{T}} & 0 & 0 & 0\\ 0 & 0 & \overline{C}^{\mathrm{T}} & 0 & 0\\ P\overline{A}_{2d} & 0 & 0 & P\overline{B}_1 & P\overline{B}_2\\ 0 & 0 & 0 & 0 & 0\\ -Q_3 & 0 & 0 & 0 & 0\\ 0 & -I & 0 & \overline{D} & 0\\ 0 & 0 & -I & 0 & \overline{D}\\ 0 & \overline{D}^{\mathrm{T}} & 0 & -\gamma^2 I & 0\\ 0 & 0 & \overline{D}^{\mathrm{T}} & 0 & -\gamma^2 I \end{array}\right] < 0 \tag{20}$$

Pre-multiplying and post-multiplying the left hand side of (20) by $diag\{P^{-1},P^{-1},P^{-1},P^{-1},P^{-1},I,I,I,I\}$ and set

$$\widetilde{P}=P^{-1}, \widetilde{Q}_k=P^{-1}Q_kP^{-1}(k=1,2,3) \tag{21}$$

we have

$$\left[\begin{array}{cccc} -\widetilde{P}+\sum_{k=1}^{3}\widetilde{Q}_k & 0 & \widetilde{P}\overline{A}_1^{\mathrm{T}} & 0\\ 0 & -\widetilde{Q}_1 & \widetilde{P}\overline{A}_2^{\mathrm{T}} & 0\\ \overline{A}_1\widetilde{P} & \overline{A}_2\widetilde{P} & -\widetilde{P} & \overline{A}_{1d}\widetilde{P}\\ 0 & 0 & \widetilde{P}\overline{A}_{1d}^{\mathrm{T}} & -\widetilde{Q}_2\\ 0 & 0 & \widetilde{P}\overline{A}_{2d}^{\mathrm{T}} & 0\\ \overline{C}\widetilde{P} & 0 & 0 & 0\\ 0 & \overline{C}\widetilde{P} & 0 & 0\\ 0 & 0 & \overline{B}_1^{\mathrm{T}} & 0\\ 0 & 0 & \overline{B}_2^{\mathrm{T}} & 0 \end{array}\right. \tag{22}$$

$$\left.\begin{array}{ccccc} 0 & \widetilde{P}\overline{C}^{\mathrm{T}} & 0 & 0 & 0\\ 0 & 0 & \widetilde{P}\overline{C}^{\mathrm{T}} & 0 & 0\\ \overline{A}_{2d}\widetilde{P} & 0 & 0 & \overline{B}_1 & \overline{B}_2\\ 0 & 0 & 0 & 0 & 0\\ -\widetilde{Q}_3 & 0 & 0 & 0 & 0\\ 0 & -I & 0 & \overline{D} & 0\\ 0 & 0 & -I & 0 & \overline{D}\\ 0 & \overline{D}^{\mathrm{T}} & 0 & -\gamma^2 I & 0\\ 0 & 0 & \overline{D}^{\mathrm{T}} & 0 & -\gamma^2 I \end{array}\right] < 0 \tag{23}$$

Furthermore, partition $\widetilde{P}$ and $\widetilde{P}^{-1}$ as follows

$$\widetilde{P}=\left[\begin{array}{cc} Y & P_{12}\\ P_{12}^{\mathrm{T}} & P_{22}\end{array}\right], \widetilde{P}^{-1}=\left[\begin{array}{cc} X & \widehat{P}_{12}\\ \widehat{P}_{12}^{\mathrm{T}} & \widehat{P}_{22}\end{array}\right] \tag{24}$$

where $Y, X, P_{12}, \widehat{P}_{12} \in R^{n\times n}$ and $P_{12}\widehat{P}_{12}^{\mathrm{T}}=I-YX$.

Let

$$J=\begin{bmatrix} X & I \\ \widehat{P}_{12}^{\mathrm{T}} & 0 \end{bmatrix}, \widetilde{J}=\begin{bmatrix} I & Y \\ 0 & P_{12}^{\mathrm{T}} \end{bmatrix} \quad (25)$$

then $J^{\mathrm{T}}\widetilde{P}=\widetilde{J}^{\mathrm{T}}, J^{\mathrm{T}}\widetilde{P}J=\begin{bmatrix} X & I \\ I & Y \end{bmatrix}>0.$

Therefore, pre-multiplying and post-multiplying Equation (23) by $diag\{J^{\mathrm{T}}, J^{\mathrm{T}}, J^{\mathrm{T}}, J^{\mathrm{T}}, J^{\mathrm{T}}, I, I, I, I\}$ and $diag\{J, J, J, J, J, I, I, I, I\}$, then by $diag\{I, \widetilde{Y}, I, \widetilde{Y}, I, \widetilde{Y}, I, \widetilde{Y}, I, \widetilde{Y}, I, I, I, I\}$, through calculation, it follows the LMI (17), where

$$\begin{aligned} Z &= -C_f P_{12}^{\mathrm{T}}\widetilde{Y}, \overline{Z}_k=\widehat{P}_{12}B_{f_k} \\ \widehat{Z}_k &= \widehat{P}_{12}A_{f_k}P_{12}^{\mathrm{T}}\widetilde{Y}(k=1,2) \end{aligned} \quad (26)$$

If the LMI (17) is feasible, the filtering system matrices $A_{f_k}, B_{f_k}(k=1,2), C_f$ and D_f can be derived from equation (26). Furthermore, the transfer function of filter (11),(12) can be turned into

$$\begin{aligned} & G_f(z_1,z_2)=-Z\widetilde{Y}^{-1}(P_{12}^{\mathrm{T}})^{-1}[z_1z_2I-z_1\widehat{P}_{12}^{-1} \\ & \times\widehat{Z}_1\widetilde{Y}^{-1}(P_{12}^{\mathrm{T}})^{-1}-z_2\widehat{P}_{12}^{-1}\widehat{Z}_2\widetilde{Y}^{-1}(P_{12}^{\mathrm{T}})^{-1}]^{-1} \\ & \times\left(z_1\widehat{P}_{12}^{-1}Z_1+z_2\widehat{P}_{12}^{-1}Z_2\right)+D_f \\ = & -Z\left[z_1z_2\widehat{P}_{12}P_{12}^{\mathrm{T}}\widetilde{Y}-z_1\overline{Z}_1-z_2\overline{Z}_2\right]^{-1} \\ & \times(z_1Z_1+z_2Z_2)+D_f \\ = & C_f\left[z_1z_2I-z_1A_{f1}-z_2A_{f2}\right]^{-1} \\ & \times(z_1B_{f1}+z_2B_{f2})+D_f \end{aligned} \quad (27)$$

where $A_{f_k}, A_{f_{kd}}, B_{f_k}(k=1,2), C_f$ and D_f are given by (18), i.e. H_∞ filtering problem for system (8),(10) is resolved. This completes the proof.

Remark 3 Theorem 1 presents the approach of variable substitution to filtering design of 2-D state-delayed systems in terms of LMI. To get filtering system matrices, similar to 1-D systems, we have to linearize matrices inequality (20) using congruence transformation based on the partition of $\widetilde{P}$ and the choice of J.

Remark 4 Though based on the same bounded real lemma as that proposed in [16], the result of robust H_∞ filter synthesis in Theorem 1 is different from [16]. The cause is that in the proof of Lemma 1 in [16], an arbitrary matrix H is introduced, which relaxes the result. At the same time, congruence transformation matrix Ω is diagonal matrix comparing with the choice of J in Theorem 1. Therefore, the result obtained in Theorem 1 is less conservative than that proposed by Theorem 4 in [16].

In Theorem 1, γ is regarded as given. However, (17) is still a LMI when γ is also a variable. Thus, it is possible to formulate the following convex optimization problem to find a filter with the smallest H_∞ norm.

Problem 1

$$\begin{aligned} & \mathit{min}\ \delta \\ X \quad > \quad & 0, \widetilde{Y}>0, J_{Q_k}>0(k=1,2,3), \mathit{and LMI\ (17)} \end{aligned}$$

applying *mincx* in Matlab Toolbox, where $\gamma=\sqrt{\delta}$. Accordingly, the optimization problem is also less convenience than that proposed by Remark 6 in [16].

4 NUMERICAL EXAMPLE

We will demonstrate the design of 2-D H_∞ filter for a stationary random field in image processing using LMI approach proposed in Problem 1 is effective and less conservative than the existing result proposed in [16].

It is known that the stationary random field can be modeled as the following 2-D system [18]:

$$\begin{aligned} \eta(i+1,j+1) &= a_1\eta(i+1,j)+a_2\eta(i,j+1) \\ &\quad -a_1a_2\eta(i,j)+\omega_1(i,j) \end{aligned} \quad (28)$$

where $\eta(i,j)$ is the state of the random field at spacial coordinate $(i,j), a_1^2<1$ and $a_1^2<1$ as a_1 and a_2 are, respectively, the horizontal and vertical correlations of the random field.

Now, we consider the influence of time delays to system (28) and introduce two terms $\eta(i+1,j-d_1)$ and $\eta(i-d_2,j+1)$ in (28) following that

$$\begin{aligned} \eta(i+1,j+1) &= a_1\eta(i+1,j)+a_2\eta(i,j+1) \\ &\quad +a_3\eta(i+1,j-d_1) \\ &\quad +a_4\eta(i-d_2,j+1) \\ &\quad -a_1a_2\eta(i,j)+\omega_1(i,j) \end{aligned} \quad (29)$$

where $a_3^2<1$ and $a_4^2<1$ as a_3 and a_4 are also, respectively, the horizontal and vertical correlations of the random field.

Denote $x^{\mathrm{T}}(i,j)=\left[\eta^{\mathrm{T}}(i,j+1)-a_2\eta^{\mathrm{T}}(i,j) \quad \eta^{\mathrm{T}}(i,j)\right]$, and assume that the measurement output is given by

$$y(i,j)=\begin{bmatrix} 3 & 1 \end{bmatrix}x(i,j)+\omega_2(i,j)$$

where ω_2 is the measurement noise. The signal to be estimated is $z(i,j)=0.5\eta(i,j)$.

It is easy to know that the 2-D system can be converted to the 2-D FM LSS model (1) or (8) with $\omega^{\mathrm{T}}=\begin{bmatrix} \omega_1^{\mathrm{T}} & \omega_2^{\mathrm{T}} \end{bmatrix}$ and

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & 0 \\ 1 & a_1 \end{bmatrix}, A_2=\begin{bmatrix} a_2 & 0 \\ 0 & 0 \end{bmatrix} \\ A_{1d} &= \begin{bmatrix} a_3 & a_1a_3 \\ 0 & 0 \end{bmatrix}, A_{2d}=\begin{bmatrix} a_4 & a_1a_4 \\ 0 & 0 \end{bmatrix} \\ B_1 &= 0, B_2=\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, C=\begin{bmatrix} 3 & 1 \end{bmatrix} \\ D &= \begin{bmatrix} 0 & 1 \end{bmatrix}, L=\begin{bmatrix} 0 & 0.5 \end{bmatrix} \end{aligned} \quad (30)$$

Let $a_1=0.2, a_2=0.3, a_3=0.15, a_4=0.03$ and given $d_1=5, d_2=6$, by solving Problem 1, the minimum H_∞ norm bound for this example is $\gamma_{opt}=0.2519$, with the following obtained matrices (for simplicity, here, we only present some items needed for the calculation of filter matrices):

$$\begin{aligned} X &= \begin{bmatrix} 0.0458 & -0.00019 \\ -0.00019 & 0.5589 \end{bmatrix} \\ Y &= \begin{bmatrix} 0.016786 & 0.00039 \\ 0.00039 & 0.002616 \end{bmatrix} \\ Z &= \begin{bmatrix} -0.02295 & -0.5075949 \end{bmatrix} \\ \overline{Z}_1 &= \begin{bmatrix} 0.0000565657 & -0.164425 \end{bmatrix}^{\mathrm{T}} \\ \overline{Z}_2 &= \begin{bmatrix} -0.0049067 & -0.00000278 \end{bmatrix}^{\mathrm{T}} \end{aligned}$$

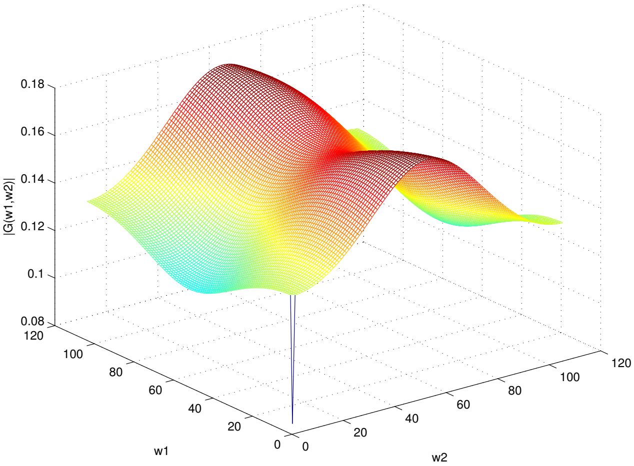

Fig. 1 Maximum singular values plot of the transfer function matrix of the filtering error system by substituting (31) to (30).

$$\widehat{Z}_1 = \begin{bmatrix} 0.00002356 & -0.0000184 \\ -0.0550808 & 0.04980843 \end{bmatrix}$$
$$\widehat{Z}_2 = \begin{bmatrix} 0.001498 & 0.004967 \\ 0.0000897 & 0.0000048 \end{bmatrix}$$

Thus, the system matrices of H_∞ filter (11),(12) can be derived from $P_{12}\widehat{P}_{12}^{\mathrm{T}} = I - YX$ and (26) as

$$\begin{aligned} A_{f_1} &= \begin{bmatrix} 0.00118 & -0.001165 \\ 0.0990 & -0.0895 \end{bmatrix} \\ A_{f_2} &= \begin{bmatrix} -0.0516 & -0.171134 \\ -0.0002 & -0.000188 \end{bmatrix} \\ B_{f_1} &= \begin{bmatrix} 0.0040 & 0.2956 \end{bmatrix}^{\mathrm{T}} \\ B_{f_2} &= \begin{bmatrix} 0.1691 & 0.00018 \end{bmatrix}^{\mathrm{T}} \\ C_f &= \begin{bmatrix} 0.02295 & 0.5076 \end{bmatrix}, D_f = -0.0082 \quad (31) \end{aligned}$$

Finally, Fig. 1 shows the maximum singular value plots of the filtering error system by connecting the filter (31) to the original system (30). In the figure, the griddings denote the obtained H_∞ disturbance attenuations and its maximum value is 0.1798, which is below 0.2519, showing the effectiveness of our filtering design procedure for a stationary random field in image processing.

With the technique proposed in [16], one can obtain the minimum H_∞ disturbance attenuation level bound $\gamma_{opt} = 0.9059$. It is shown that the filtering design proposed in this paper is less conservative than that obtained in reference [16].

5 CONCLUSION

This paper considers H_∞ filtering problem for 2-D discrete systems with state delays. In terms of the existing bounded real lemma of 2-D state-delayed systems, the corresponding H_∞ filter is designed to assure the asymptotic stability and H_∞ performance γ of 2-D state-delayed systems through LMI's feasibility. In addition, an optimization problem for solving the minimum H_∞ performance is given. A numerical example proves the effectiveness and less conservatism of our result.

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